Basic Calculus Vocabulary | APTS 7

Useful word, Useful sentences and pronunciation help

(see the International Phonetic Alphabet on the last page)

■ **Numbers**

- $x^2 - 3$ reads "x ***squared minus*** three" /ˈmaɪnəs/. The ***opposite*** of 3 is -3 "***negative*** three"
- 3×5 reads "3 ***times*** 5".
- x^3 reads "x ***to the third*** power" or simply "x ***to the third***" or "x **cubed**".
- $x \leqslant 4$ reads "x is ***less than or equal to*** 4".
- $x \geqslant 3.2$ reads "x is ***greater than or equal to*** 3 ***point*** 2".
- ☠ $x > 0$ reads "x is ***positive***" /ˈpɒzɪtɪv/ whereas $x \geqslant 0$ reads "x is ***nonnegative***".
- 0, 2 , 4, 6, ...are ***even*** numbers whereas 1, 3, 5, 7.. are ***odd*** numbers.
- The ***interval*** /ˈɪntərvəl/ $[a, b)$ is the set of all real numbers x such that $a \leqslant x < b$.

■ **Functions** /ˈfʌŋkʃən/

- $f(x)$ reads "f *of* x".
- If we ***substitute 2 for*** x in $f(x)$ we get $f(2)$. /ˈsʌbstɪˌtut, -ˌtyut/
- Some quantities can be ***estimated*** /ˈɛstəˌmeɪ tid/ from a ***graph of the function*** /græf, grɑf/
- If f is defined by $f: \begin{array}{ccc} [2\,;\,5] & \rightarrow & \mathbb{R} \\ x & \mapsto & x+1 \end{array}$ then $[2\,;5]$ is the ***domain***, $\mathbb{R}$ is the codomain and [3 ; 6] is the range: The ***range*** /reɪndʒ/ is the set of all possible values of $f(x)$ as x varies throughout the domain.
- If f satisfies $f(-x) = f(x)$ for all x in its domain, then f is called an ***even*** function. The graph of an even function is ***symmetric with respect to the y-axis***. /sɪˈmɛtrɪk/
- If f satisfies $f(-x) = -f(x)$ for all x in its domain, then f is called an ***odd*** function. The graph of an odd function is ***symmetric about the origin***. If we already have the graph of f for x>=0, we can obtain the entire graph by rotating through 180 about the origin.
- ☠ Functions of the form $f(x) = mx + p$ are called ***linear***. There is a good reason for this: The graph of these functions are ***(straight) lines***. m is the ***slope*** of the line and p is its ***y-intercept*** (=intersection with the y-axis).

■ **Limit** /ˈlɪmɪt/

- $\lim_{x \to a} f(x)$ reads "*limit of* f *of* x ***as*** x ***approaches*** a"
- $\lim_{\substack{x \to a \\ x > a}} f(x)$ reads "*limit of* f *of* x ***as*** x ***approaches*** a ***from the right***"
- We will study ***vertical*** and ***horizontal asymptotes***/ˈvɜrtɪkəl/ /ˌhɔrəˈzɒntl, ˌhɒr-/ /ˈæsɪmˌtoʊt/
- f is ***continuous at*** a number a if $\lim_{x \to a} f(x) = f(a)$ /kənˈtɪnyuəs/

■ **Derivative** /dɪˈrɪvətɪv/

- You may use the derivative to determine whether a function ***is increasing*** or ***decreasing***.
- The ***tangent line*** to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

■ **Integrals** /ˈɪntɪgrəl/

- $\int_a^b f(x)\,dx$ reads *"integral of f of x from a to b"*

Problems

♠ Exercise 1. Putting it all together (not too hard)

Sketch the graph of a function $f(x)$ which has **all** of the following properties:

1. $\lim_{x\to 2^-} f(x) = -\infty$

2. $\lim_{x\to 2^+} f(x) = \infty$

3. $\lim_{x\to -\infty} f(x) = 0$

4. $f(-2) = 2$

5. $f(5) = 1$

6. $f(0) = 0$

7. $f'(x) > 0$ if $x < -2$ or $x > 5$

8. $f'(x) < 0$ if $-2 < x < 2$ or $2 < x < 5$

9. $f'(5) = 0$

10. $f'(-2) = 0$

♠ Exercise 2. Putting it all together (quite hard)

Sketch the graph of a function $f(x)$ which has **all** of the following properties:

1. f has domain $(-\infty, -2) \cup (-2, \infty)$

2. f has range $(-5, \infty)$

3. The graph of f has a vertical asymptote at $x = -2$

4. $\lim_{x\to -\infty} f(x) = 2$

5. $\lim_{x\to \infty} f(x) = -5$

6. $\lim_{x\to 3} f(x) = 2$

7. f is discontinuous at $x = 3$

8. $f'(x) > 0$ on $(-\infty, -2)$

9. $f'(x) < 0$ on $(-2, 3)$

10. $f'(4)$ is not defined, but f is continuous at 4

♠ Exercise 3. Asymptotes /ˈæsɪmˌtoʊt/

Describe all vertical and horizontal asymptotes of $f(x) = \dfrac{3x^2+4x+5}{\sqrt{16x^4-81}}$

♠ Exercise 4. Asymptotes /ˈæsɪmˌtoʊt/

Sketch the graph of an *odd* function with $y=1, x=-4$ and $x=-1$ among its asymptotes.

♠ Exercise 5. Continuity

$g(x)=\begin{cases} x^2-c^2 & \text{if } x<4 \\ cx+20 & \text{if } x\geq 4 \end{cases}$. Find the constant c that makes g continuous on $(-\infty, +\infty)$ and for this value of c, sketch the graph of g and determine whether there is a line tangent to $y=g(x)$ at $(4, g(4))$.

♠ Exercise 6.

A Tibetan monk leaves the monastery at 7:00 AM and takes his usual path to the top of the mountain, arriving at 7:00 PM. The following morning, he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 PM. Use the Intermediate Value Theorem to prove that there is a point on the path that the monk will cross exactly at the same time on both days.

/tɪˈbɛtn/ /ˈmɒnəˌstɛri/

♠ Exercise 7. Optimization and fencing [Quite easy]

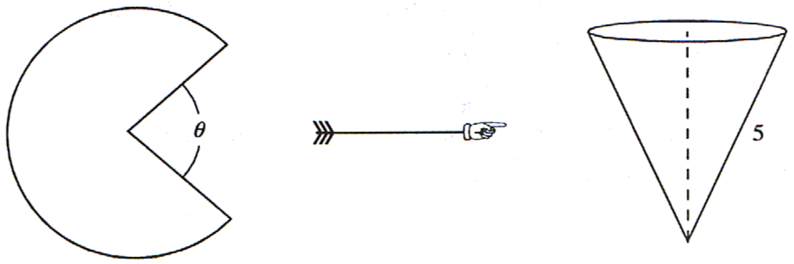
A farmer has a 2 400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

♠ **<u>Exercise 8</u>.** Calculus in England, Optimization [Not so easy]

No summer visit to London is complete without having lunch on sunny Goodge Street. There, for a mere pound coin, you can purchase the best fish and chips you've ever tasted from any one of a number of friendly street vendors. One of the reasons that the prices are so reasonable is that they give you no silverware, nor even a plate; they just roll up a piece of paper into a cone, and toss your food in. (They DO give you a little packet of vinegar, though.)

Neither a long, skinny cone nor a wide fat cone would hold enough fish and chips to make anybody happy. The vendors must be trained to roll a cone of the perfect size. Many students in the United States will never get a chance to see sunny Goodge Street, but by trying to solve the vendors' problem of optimizing the volume of a cone, we can feel as if we are there now.

For modeling purposes, assume that the piece of paper is a circle of radius 5, and that we are cutting a wedge out of it whose central angle is θ.



Find the value of θ that maximizes the volume of the cone. What is the maximum volume of the cone?

Sources: http://dictionary.reference.com, and Stewart Calculus textbook.

International Phonetic Alphabet : Pronunciation Key

In order to check the pronunciation of any word, you can go to http://www.reference.com/, type the word and then click on the microphone icon : The word will be read aloud to you.

Alternatively, you can learn to *read* the pronunciation symbols of the International Phonetic Alphabet (IPA) using the explanations below :

Stress marks: /ˈ/ indicates primary stressed syllable follows; /ˌ/ indicates secondary stressed syllable follows, as in **news·pa·per** /ˈnuzˌpeɪ pər/and **in·for·ma·tion** /ˌɪn fərˈmeɪ ʃən/

CONSONANTS	
/b/	boy, baby, rob
/d/	do, ladder, bed
/dʒ/	jump, budget, age
/f/	food, offer, safe
/g/	get, bigger, dog
/h/	happy, ahead
/k/	can, speaker, stick
/l/	let, follow, still
/m/	make, summer, time
/n/	no, dinner, thin
/ŋ/	singer, think, long
/p/	put, apple, cup
/r/	run, marry, far, store
/s/	sit, city, passing, face
/ʃ/	she, station, push
/t/	top, better, cat
/tʃ/	church, watching, nature, witch
/θ/	thirsty, nothing, math
/ð/	this, mother, breathe
/v/	very, seven, love
/w/	wear, away
/ʰw/	where, somewhat
/y/	yes, onion
/z/	zoo, easy, buzz
/ʒ/	measure, television, beige

VOWELS	
/æ/	apple, can, hat
/eɪ/	aid, hate, day
/ɑ/	arm, father, aha
/ɛər/	air, careful, wear
/ɔ/	all, or, talk, lost, saw
/aʊər/	hour
/ɛ/	ever, head, get
/i/	eat, see, need
/ɪər/	ear, hero, beer
/ər/	teacher, afterward, murderer
/ɜr/	early, bird, stirring
/ɪ/	it, big, finishes
/aɪ/	I, ice, hide, deny
/aɪər/	fire, tired
/ɒ/	odd, hot, waffle
/oʊ/	owe, road, below
/u/	ooze, food, soup, sue
/ʊ/	good, book, put
/ɔɪ/	oil, choice, toy
/aʊ/	out, loud, how
/ ʌ /	up, mother, mud
/ə/	about, animal, problem, circ

Source:

http://www.reference.com/

Teachers corner, useful sentences

- Graph a function on the board and have students call out rough estimates of the derivatives.
- Have the students work on this in groups of 3 or 4.
- It is crucial that students discuss problem 4.
- What would $f'(a)$ mean in real terms in this instance?