Épreuve de section européenne

[ ] Euclid's solution of some quadratic equations

- *Brief history of the quadratic equation*

  It is often claimed that the Babylonians (about 1600 BC) were the first to solve quadratic equations. This is an oversimplification, for the Babylonians had no notion of "equation". What they did develop was an algorithmic approach for solving problems which, in our terminology, would give rise to a quadratic equation. However all Babylonian problems had answers which were positive (more accurately unsigned) quantities since the usual answer was a length.

  In about 300 BC Euclid developed a geometrical approach which amounted to finding a length which, in our notation, was the root[1] of a quadratic equation. Although later mathematicians used it to actually solve quadratic equations, Euclid had no notion of equation, coefficients etc. but worked with purely geometrical quantities.

  Hindu mathematicians took the Babylonian methods further so that Brahmagupta (598-665 AD) gave a method which admits negative quantities. He also used abbreviations for the unknown, usually the initial letter of a colour was used, and sometimes several different unknowns occur in a single problem.

  The final, complete solution as we know it today came around 1100 AD, by another Hindu mathematician called Baskhara. Baskhara was the first to recognise that any positive number has two square roots.

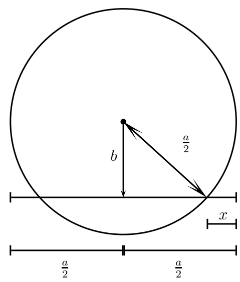
- *Euclid's method*

  Euclid's method for solving quadratic equations only applies to equations of the following type :

$$x^2 - ax + b^2 = 0 \qquad \text{with } a > 0 \text{ and } b > 0. \tag{1}$$

  It goes as follows, the picture on the right helping to follow the different steps :

  1. Draw a line of length a.
  2. Draw a line of length b perpendicular to the previous one, from its midpoint.
  3. Draw a circle centred at the end of this line, with radius $\frac{a}{2}$.
  4. The length denoted x gives the solution of the equation $(a-x)x = b^2$.



From The MacTutor History of Mathematics.

Questions

1. Which mathematician mentioned in the text was one of the first to use a letter to represent the unknown?
2. Give an equation with at least one negative root ?
3. Check that the expression in point 4 of Euclid's method is equivalent to equation (1).
4. At which condition on a and b does equation (1) admit 2 solutions ?
5. Find out the solutions of $x^2 - 5x + 4 = 0$ with the modern method. Does Euclid's method apply in this case ?
6. Draw the figure corresponding to the equation above. Measure the length called x in Euclid's method and check that it is indeed a solution of $x^2 - 5x + 4 = 0$.
7. Prove that Euclid's method is valid.

[1] root : solution

Baccalauréat, toutes séries — Session de juin 2012

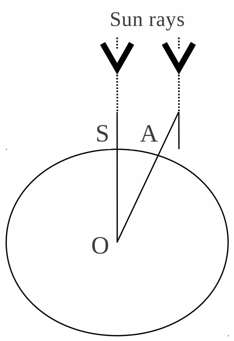
Épreuve de section européenne

[ ] Eratosthenes' measurement of the Earth

The most famous scientific achievement of Eratosthenes was his measurement of the earth. Eratosthenes improved a method that, according to Archimedes, some had tried, in order to calculate the circumference of the earth.

He observed that at Syene, at noon, at the summer solstice, the sun cast no shadow from an upright gnomon[1], while at the same moment the gnomon fixed upright at Alexandria (taken to be on the same meridian with Syene) cast a shadow corresponding to an angle between the gnomon and the sun rays of 1/50th of a complete circle or four right angles.

The sun rays are of course assumed to be parallel at the two places represented by S and A in the annexed figure. If α be the angle made at A by the sun rays with the gnomon (OA produced), the angle SOA is also equal to α, or 1/50th of four right angles.



Now the distance from S to A was known by measurement to be 5,000 stades; it followed that the circumference of the earth was 250,000 stades. Theon of Smyrna reported that Eratosthenes corrected it to 252,000 stades for some reason, perhaps in order to get a figure divisible by 60 and, incidentally, a round number of stades per one degree. On the basis that the length of a stade was 157.5 meters, 252,000 stades work out to be 24,662 miles, and the diameter of the earth to be about 7,850 miles, only about 50 miles shorter than the true polar diameter, a surprisingly close approximation, however much it owes to happy accidents in calculation.

From *A History of Greek Mathematics*, by Sir Thomas Heath

Questions

1. Assuming that the Earth is a perfect sphere, and using Eratosthenes reasoning, prove that its circumference is, indeed, 250 000 stades.
2. Explain why $\angle SOA = \alpha$. Hence deduce the value of α in degrees.
3. By taking the value given by Theon of Smyrna, calculate the round number of stades per degree.
4. Use the last part of the text to calculate the number of kilometres in one mile, rounded to 3 d.p.
5. Name two mathematicians mentioned in the text. Do you know any other mathematicians or scientists from the Antiquity ? Can you mention some of their works ?

[1]Gnomon: A stick of wood that shows the time of the day by casting its shadow on a plane surface.