

Section Européenne DNL mathématiques
Syracuse problem = Collatz conjecture

2^{nde}
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Topics: understanding an algorithm, forming a conjecture, programming

The **Syracuse problem**, also known as the **Collatz conjecture** or the **$3n+1$ conjecture** or **Ulam conjecture**, is a very simple problem of arithmetics that is still unsolved today.

It can be stated as follows :

The **Syracuse algorithm** :

$n \geq 1$ being an integer, do the following:

- If the number is *even* then divide it by two;
- If the number is *odd* then multiply it by 3 and then add 1;
- Repeat the process (which has been called "*Half Or Triple Plus One*", or **HOTPO**) indefinitely.

Useful words

- a **conjecture** /kən'dʒektʃər/ : a guess, an opinion without a proof (yet).
- An **algorithm** /'ælgə,rɪðəm/ is a sequence of instructions.
- to **conjecture** = to *form* conjectures.
- **even** /'ivən/ = pair
- **odd** /ɒd/ = impair.
- an **array** /ə'rei/ = un tableau.
- a **sequence** /'sɪkwəns/ = une suite.

1) Exploring the Syracuse problem. No calculator please!!

a) Start with 29, apply the algorithm and write down what you get after each step in the first row of the **array** below.

b) A number is assigned to each student the following way: If your first name starts with A, your number is 1; If your first name starts with B, your number is 2 ...etc. (so as usual, $A=1, B=2, C=3, D=4 \dots$ etc). Starting with this number, apply the algorithm and write down what you get after each step in the corresponding row of the array below.

c) Starting with the integer corresponding to the first letter of your last name, fill in the corresponding row of the array below.

After n steps														
	Initial number $n=0$	After 1 step $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$	$n=11$	$n=12$	$n=13$
Start with 29	29													
First name														
Last name														

You may use the empty rows to try out more numbers or to help someone with their number.

2) Conjecture: Do you notice a pattern? Can you guess what the Collatz Conjecture is?

3) Become famous¹: Find a counter example! Write a program in OpenOffice.calc or Excel that computes the *sequence* of numbers you get when you iterate the algorithm.

¹ *I feel I have to tell you that great mathematician with extremely powerful computers have failed at this so far...*

Coin profs

- The conjecture is that no matter what number you start with, *you will always eventually reach 1.*
- This is an unsolved problem!!! They don't often see one so let's get them excited about it!
- Example : *starting with n=10 the sequence is*
10 -> 5 -> 16 -> 8 -> 4 -> 2 -> 1
- If we continue the process after 1 then it *indefinitely repeats a cycle*
1 -> 4 -> 2 -> 1 -> 4 -> 2 -> 1 ...
- Syntaxe OpenOffice.calc =SI(MOD(A3;2)=0; A3/2;A3*3+1)
MOD(m; q) donne le reste de la division euclidienne de a m par q;
si(condition; valeur_alors; valeur_sinon)
Cela donne :

1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
3	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2
5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2
6	3	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2
7	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1
8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
9	28	14	7	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1
10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2
12	6	3	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
14	7	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2
15	46	23	70	35	106	53	160	80	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2
16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4
18	9	28	14	7	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2
19	58	29	88	44	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2
20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
21	64	32	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4
23	70	35	106	53	160	80	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4
24	12	6	3	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
25	76	38	19	58	29	88	44	22	11	34	17	52	26	13	40	20	10	5	16	8	4	2	1	4	2
26	13	40	20	10	5	16	8	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1

Présentation pour Euromath

- 1) Découverte de la suite de Syracuse;
- 2) Programmation du calcul des termes;
- 3) Formulation d'une conjecture.

Une occasion de voir un problème ouvert simple à comprendre.