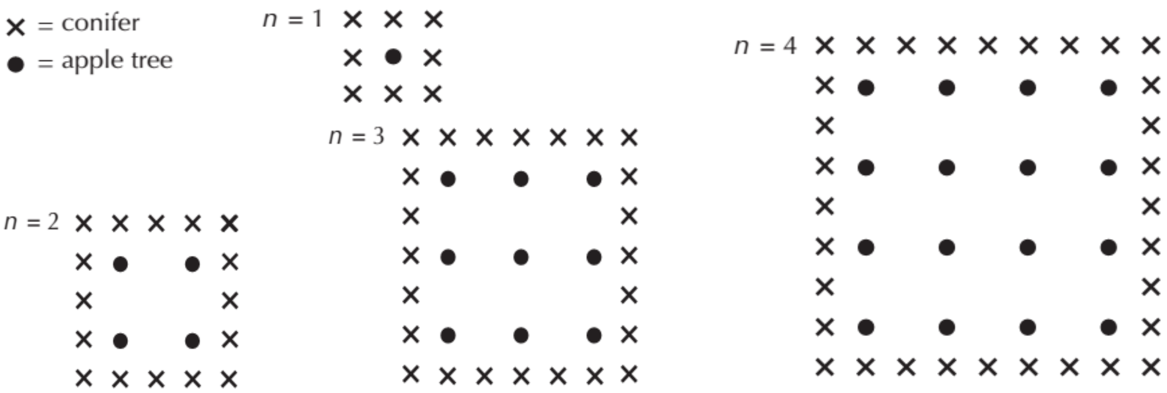
Algebra review | DNL 1ère

◯ Problem 1. Apples [Source : PISA[1]]

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.
Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:

Prounounciation corner :
- conifer /'kɒnəfər/
- orchard /'ɔrtʃərd/
- pattern /'pætərn/



1) *Complete the table:*

n	*Number of apple trees*	*Number of conifer trees*
1	1	8
2	4	
3		
4		
5		

2) There are two formulae you can use to calculate the number of apple trees and the number of conifer trees for the pattern described on the previous page:
Number of apple trees = n^2 ; Number of conifer trees = 8n where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifer trees. Find the value of n and show your method of calculating this.

◯ Problem 2. Lichen [Source : PISA]

A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plants, called lichen, start to grow on the rocks.
Each lichen grows approximately in the shape of a circle. The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

$$d = 7 \times \sqrt{t-12} \quad \text{for} \quad t \geqslant 12$$

Prounounciation corner :
- lichen /'laɪkən/
- tiny /'taɪni/
- shape /ʃeɪp/
- relationship /rɪ'leɪʃənʃɪp/
- diameter /daɪ'æmɪtər/

where d represents the diameter of the lichen in millimetres, and t represents the number of years after the ice has disappeared.

1) Using the formula, calculate the diameter of the lichen, 16 years after the ice disappeared.
Show your calculation.

2) Ann measured the diameter of some lichen and found it was 35 millimetres.
How many years ago did the ice disappear at this spot?
Show your calculation.

[1] The **Programme for International Student Assessment** (**PISA**) is a worldwide study by the Organisation for Economic Co-operation and Development (OECD) in member and non-member nations of 15-year-old school pupils' scholastic performance on mathematics, science, and reading. It was first performed in 2000 and then repeated every three years. It is done with view to improving education policies and outcomes.

ALGEBRA Useful words and concepts

- If the terms have ***a factor in common***, you you ***factor it out***. For example, in order to ***factor*** $2x^3-5x$, you can ***factor x out***.
- In order to ***expand*** the polynomial $x(2x^2-3)$, you can ***distribute x through the parenthesis***.
- In the equation $2x+7-2x=6$, the terms $2x$ et $-2x$ ***cancel out***.
- ***Sign rule*** /sain/ : "*A negative times a positive is a negative.*"
- "*If the product of two quantities is equal to zero than one or both of them have to be equal to zero*".
- Multiplying both sides of an inequality by a negative number reverses the direction of the inequality.

◯ Exercise 3. Learning the vocabulary from an example

Based on a Khan Academy video :
https://www.khanacademy.org/math/algebra/linear_inequalities/inequalities/v/solving-inequalities

1) Watch the video (note that americans denote $8+\frac{1}{3}$ by $8\frac{1}{3}$).

2) Solve $\frac{4}{5}<-3t-\frac{2}{5}$ and be ready to explain the steps at the board. You may use the video as a model. Write down the useful sentences and words in your little notebook, practice before you give your speech.

◯ Exercise 4. Learning the vocabulary from an example

1) Read the following solved problem

Problem: Solve $x^2-3x=10$.

Solution : First, subtract 10 from both sides to get $x^2-3x-10=0$ *and then factor the resulting polynomial on the left-hand side to get* $(x-5)(x+2)=0$ *. Since the product (x-5)(x+2) can be zero only if one (or both) of its factors are zero, it follows that the solutions are x=5 (which makes the first factor zero) and x=-2 (which makes the second factor zero).*

2) a) Expand $(2x+1)(5-x)$; **b)** Solve $-2x^2+9x=-5$

3) a) Expand $(2-3x)(3-2x)$; **b)** Solve $9x^2-16x+6=3x(x-1)$

◯ Exercise 5. The relationship between the Celsius ans Fahrenheit temperature scale is given by $C=\frac{5}{9}(F-32)$ where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit. What interval on the Celsuis scales correspods to the temperature range $50\leqslant F\leqslant 95$?

◯ Exercise 6. The relationship between the Celsius ans Fahrenheit temperature scale is given by $C=\frac{5}{9}(F-32)$ where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit. What interval on the Celsuis scales correspods to the temperature range $20\leqslant C\leqslant 30$?

◯ Exercise 7. Monthly salary

A pharmaceutical salesperson receives a monthly salary of $2500 plus a commission of 7% of sales.

1) Write an equation for the salesperson monthly salary in terms of the number of monthly sales *S*.

2) What is the amount of sales the salesperson needs to make in order to earn a $2900 monthly salary?

◯ Exercise 8. Hourly wage

A microchip manufacturer pays its assembly line workers $9.50 per hour for a eight-hour workday. In addition, workers receive a piecework rate of $0.15 per unit produced.

1) Write an equation of the hourly wage in terms of the number of unit *x* produced per hour.

2) Bob worked for 8 hours last monday and he made 136 $ on that day. On average, how long does it take him to produce one piece?

3) Challenge question: If a person works more than 8 hours on a given day the extra hours are paid 50% more. Write an equation of the hourly wage in terms of whatever variables you need.

Exercise 9. Using the best suited expression

Let f be the function defined on $\mathbb{R}$ by $f(x) = 4(x+1)^2 - 9$ and let $\mathcal{C}_f$ be its graph.

Useful words		
Product	expand →	Sum
$k \times (a+b)$		$ka + kb$
$(a+b)(a-b)$	← factor	$a^2 - b^2$

1) a) Expand $f(x)$; **b)** Factor $f(x)$.

2) From the previous question, we now have 3 expressions for f: The *expanded form*, the *factored form* and the *standard form*.

a) Make sure you use the expression best suited to the problem to compute the following numbers: $f(0)$; $f(-1)$; $f\left(\frac{1}{2}\right)$; $f(2)$

b) You have 3 expressions for f. Make sure you use the one best suited to the problem to solve the following equations: $f(x) = 0$; $f(x) = -16$; $f(x) = -5$; $f(x) = -9$.

c) Find the points where $\mathcal{C}_f$ intersects the x-axis.

d) Find the points where $\mathcal{C}_f$ intersects the y-axis.

♠ Problem 10. A solved mixture problem.

Problem : How many liters of 20% alcohol solution should be added to 40 liters of a 50% alcohol solution to make a 30% solution?

Solution : Let x be the quantity of the 20% alcohol solution to be added to the 40 liters of a 50% alcohol. Let y be the quantity of the final 30% solution. Hence x + 40 = y

- *We shall now express mathematically that the quantity of alcohol in x liters plus the quantity of alcohol in the 40 liters is equal to the quantity of alcohol in y liters. But remember the alcohol is measured in percentage term. 20% x + 50% * 40 = 30% y*
- *Substitute y by x + 40 in the last equation to obtain.20% x + 50% * 40 = 30% (x + 40)*
- *Change percentages into fractions: 20 x / 100 + 50 * 40 / 100= 30 x / 100 + 30 * 40 / 100*
- *Mutliply all terms by 100 to simplify: 20 x + 50 * 40 = 30 x + 30 * 40*
- *Solve for x : x = 80 liters*

80 liters of 20% alcohol is be added to 40 liters of a 50% alcohol solution to make a 30% solution.

Problem 11. Another mixture problem.

Harame wants to make a 100 ml of 5% alcohol solution mixing a quantity of a 2% alcohol solution with a 7% alcohol solution. What are the quantities of each of the two solutions (2% and 7%) he has to use?

Hint: You may want to read the solution to the above problem before you write down your solution.

Problem 12.

1) Let A(1, 3) and B(-5, 2). Let I be the midpoint of the line segment [AB]. Determine the coordinates of I.

2) Let A(1, 3) and B(-5, 2). B is the midpoint of the line segment [AC]. Determine the coordinates of C.

Midpoint Formula

To find the *midpoint* of the *line segment* joining the points $A(x_A, y_A)$ and $B(x_B, y_B)$, you can simply find the average values of the respective coordinates of the two endpoints, namely:

$$\text{Midpoint} = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

Notes from last year

Useful words

- When you ***square*** x, you get x^2 (Read x *squared*).
- When you take the ***inverse*** of x, you get $\frac{1}{x}$ (Read "1 *over* x")..
- When you take the ***square root*** of x, you get $\sqrt{x}$.

Useful words

- "x is ***positive***" means $x > 0$.
- "x is ***nonnegative***" means $x \geqslant 0$.
- $x \geqslant 0$ reads "x is ***greater than or equal to*** zero."
- $x > 0$ reads "x is ***greater than*** zero"
- $x < 0$ reads "x is ***less than*** zero".