Épreuve de section européenne

The amazing number 6174

At first glance, the number 6174 looks like any other number. Perhaps this is what makes this unassuming number so remarkable. In 1949, Indian mathematician D. R. Kaprekar, discovered the mysterious beauty of 6174 after devising a process that we now know as Kaprekar's operation.

The algorithm:

- start with any four-digit number such that not all of its digits are equal;
- re-arrange the digits in ascending order to make one four-digit number and in descending order to make a second four-digit number (don't forget the possible zeros);
- subtract the smaller number from the larger number;
- repeat until you get the same number for every iteration.

You'll always end up with 6174 within seven or fewer iterations. After you get that number, you can repeat this operation until the end of time, you will always get the same answer: 6174.

For example, if we choose the year 2011 as our starting four-digit number, we end up with:

1° $2110 - 0112 = 1998$; **2**° $9981 - 1899 = 8082$; **3**° $8820 - 0288 = 8532$; **4**° $8532 - 2358 = 6174$: it took just four iterations to reach 6174. From this point onwards, this operation can be repeated and it will always yield the same answer: $7641 - 1467 = 6174$.

What does this mean? Well, nothing really. It's just one of those marvellous things that we can simply enjoy for what it is. But I'll bet those of you who like indulging in bar bets will enjoy this maths trick.

Adapted from GrrlScientist's column, *The Guardian*, 12 December 2011.

Questions

1. Why is the algorithm not working when all the digits are equal?
2. Starting with year 2011, you need four iterations. Is year 2012 longer or quicker? And what about year 1949, which is the year of Kaprekar's birth?
3. We can adapt Kaprekar's algorithm to 3-digit numbers (whose digits are not all equal). Try several 3-digit numbers; what is your conjecture?
4. Prove your conjecture of question 2 (Hint: the digits of the number are a, b and c; you can suppose without loss of generality that a is the lesser and c is the greater.)
5. You probably noticed that all the numbers generated after the first iteration in question 1 are divisible by 9. Prove it.
6. The original text was subtitled: because not everything has to be useful to be interesting! Could you give several examples of useless things that are interesting to you, both in mathematics and in everyday life?