Source : *Fermat's last theorem* by Simon Singh

Science is operated according to the judicial system. A theory is assumed to be true if there is enough evidence to prove it 'beyond reasonable doubt'. On the other hand mathematics does not rely on evidence from fallible experimentation but, it is built on infallible logic. This is demonstrated by the problem of the 'mutilated chessboard', illustrated in the figure below.
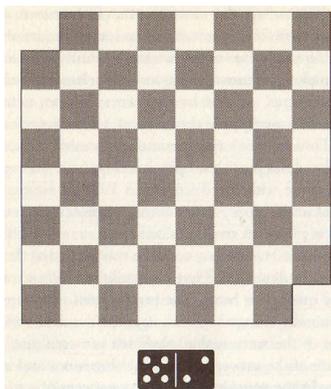
We have a chessboard with the two opposing corners removed so that there are only 62 squares remaining. We now take dominoes shaped such that each domino covers exactly two squares. The question is: Is it possible to arrange the dominoes so that they cover the 62 squares on the chessboard?

There are two approaches to the problem:

(1) The scientific approach
The scientist would try to solve the problem by experimenting, and after trying out a few dozen arrangements would discover that they all fail. Eventually the scientist believes that there is enough evidence to say that the board cannot be covered. However the scientist can never be sure that this is truly the case because there might be some arrangement that hasn't been tried which might do the trick. There are millions of different arrangements and it is only possible to explore a small fraction of them. The conclusion that the task is impossible is a theory based on experiment, but the scientist will have to live with the prospect that one day the theory may be overturned.

(2) The mathematical approach
The mathematician tries to solve the problem by developing a logical argument which will derive a conclusion which is undoubtedly correct and which will remain unchallenged forever. One such argument is the following:

- The corners which were removed from the chessboard were both white. Therefore there are now 32 black squares and only 30 white squares.
- Each domino covers two neighbouring squares, and the neighbouring squares are always different in colour, i.e. one black and one white.
- Therefore, no matter how arranged, the first 30 dominoes laid on the board must cover 30 white squares and 30 black squares.
- Consequently, this will always leave you with one domino and two black squares remaining.
- But remember all dominoes cover two neighbouring squares, and neighbouring squares are opposite in color. However the two squares remaining are the same color and so they cannot both be covered by the one remaining domino. Therefore, covering the board is impossible!

This proof shows that every arrangement of dominoes will fail to cover the chessboard!

■ **Questions**

1) This text mentions scientists, as opposed to mathematicians. What kind of scientists do you know?

2) How is science similar to the judicial system?

3) In order to solve the mutilated chessboard problem, will a scientist explore all the possible arrangements? will a mathematician explore all the possible arrangements?

4) Explain in your own word the difference between truth in math and truth in science. In each case, what is the main tool to reach the 'truth'?

■ **Check your progress**: The dominoes mentioned in all the questions below are shaped such that each domino covers exactly two squares.

1) We have a chessboard with one corner removed. Is it possible to arrange the dominoes so that they cover the mutilated chessboard?

2) We have a chessboard with two adjacent corners removed. Is it possible to arrange the dominoes so that they cover the mutilated chessboard?

3) We have a chessboard with four black squares removed. Is it possible to arrange the dominoes so that they cover the mutilated chessboard?

4) Create your own!

Classe : Première, après avoir fait des "Vrai ou faux ?" en français, ou quand ils commencent à demander comment on peut démontrer que quelque chose n'existe pas (lorsque l'on évoque Galois, dans le second degré)

Objectifs :
- Comprendre le caractère spécifique de la preuve en mathématiques par rapport aux autres sciences.
- Comprendre comment on peut démontrer qu'une solution existe ou n'existe pas…et le pratiquer !
- Travailler sur un texte authentique (extrait de livre).

Déroulement:
- D'abord leur donner l'énoncé et les laisser y réfléchir et expérimenter.
- Leur distribuer le texte et les questions.
- Demander aux plus rapides d'inventer des questions dans la même veine et de les échanger entre eux.

Chers collègues : Pour me faire parvenir vos commentaires sur ce document et/ou échanger des idées sur la DNL, vous pouvez me contacter à lhelmeg@yahoo.com.